REPORT

PROPOSED COST OF SERVICE GAS HEDGE AGREEMENT

BETWEEN

BLACK HILLS NEBRASKA AND BLACK HILLS UTILITY HOLDINGS, INC.

Provided for the Consideration of

NEBRASKA PUBLIC SERVICE COMMISSION

Prepared By

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March 22, 2016

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INTRODUCTION

Reaching back decades, Black Hills Corporation (BHC) provides natural gas and electricity services within retail markets. BHC is a diversified energy company, with natural gas and electric utility operations combined with non-regulated activities focused on power generation, coal mining, and oil and gas exploration. The broad range of BHC's business activities sets it apart from other combination gas and electric utilities. While several entities focused on gas distribution also have substantial subsidiary operations, such operations are predominantly retail services, competitive supply for larger customers within retail markets, and intrastate pipelines of comparatively modest scale.

The gas distribution business is fairly straightforward: natural gas is purchased from producers under short- and long-term[2] contractual arrangements, which often include short- and medium-term hedges such as storage and financial swap agreements. Gas purchases by distributors – e.g., the various transactions – are carried out under these arrangements. Typically, the expenditure bundle associated with these transactions is then recovered from retail consumers under purchased gas adjustment (PGA) clauses.

Black Hills/Nebraska Gas Utility (Black Hills Nebraska or Company) is a subsidiary of Black Hills Utility Holdings LLC (BHUH), BHC's largest subsidiary organization, generally referred to as the Utilities Group. Black Hills Utility Holdings has petitioned the regional regulatory authorities for Colorado (Public Utilities

[1] Principal Investigator and main author.

[2] The notion of *long-term* refers to contracts with a term beyond one year. It is not unusual for gas distributors to have contracts in place with gas suppliers covering several forward years, but where the prices under the contractual umbrella vary, partially or wholly, with respect to contemporary market prices.

Commission of Colorado or Colorado PUC), Iowa (Iowa Public Utilities Board or Iowa PUB), Kansas (Kansas Corporation Commission or Kansas CC), Nebraska (Nebraska Public Service Commission, Nebraska PSC, or Commission), South Dakota (South Dakota Public Utilities Commission or South Dakota PUC), and Wyoming (Wyoming Public Service Commission or Wyoming PSC) for authorization to integrate the operating results associated with gas exploration and production within the total costs used to set rates for retail gas distribution.

Along with several other elements of its petition, the Company's proposal requests the Nebraska PSC to approve a contractual arrangement, referred to as the Cost of Service Gas Hedge Agreement (COSG Agreement, COSG), between Black Hills Nebraska and BHUH. Under the COSG Agreement, the prices underlying wholesale gas purchases of Black Hills Nebraska are partially hedged, for the subscription share of Black Hills Nebraska's sales quantities covered by the COSG. The proposed contractual arrangement by Black Hills Nebraska constitutes a clear break from the longstanding business arrangements underlying gas distribution. The cost of gas of gas exploration and production is internalized within the Company's retail PGA clauses, with the potential to lower the overall cost of gas paid by retail consumers. The core argument advanced by BHE for requesting authority to undertake this step is that financial hedges of a long-term nature are either not available or prohibitively expensive due to the thinness of the market for long maturities.

DESCRIPTION OF BLACK HILLS CORPORATION

Black Hills Corporation, a South Dakota corporation, is a vertically-integrated energy company headquartered in Rapid City, South Dakota. The predecessor of Black Hills Corporation was Black Hills Power and Light Company, which was incorporated and began providing electric utility service in 1941. Black Hills Corporation began producing, selling and marketing various forms of energy through non-regulated businesses in 1956. Black Hills Corporation has ongoing oil and gas exploration and production activities.

The business operations of Black Hills Corporation include two major business groups: the utilities Group (BHUH) and the non-regulated energy group, Black Hills Non-Regulated Holdings LLC. BHE's other subsidiaries include Black Hills Power and Cheyenne Light, Fuel and Power Company (Cheyenne Light). The utilities group, is comprised of regulated Electric Utilities and regulated Gas Utilities segments. The Utilities group generates, transmits, and distributes electricity to approximately 207,200 electric customers in the states of South Dakota, Wyoming, Colorado, and Montana. For electricity services, power generation is operated by three electric utility subsidiaries including Black Hills Power, Cheyenne Light, and Colorado Electric,[3] with 2015 peak demands of 424MW, 212MW, and 392MW, respectively.

[3] A number of the generating units across the utility business group are co-owned including 184.6MW for Black Hills Power, 40MW for Cheyenne Light, and 14.5MW for Colorado Electric.

Stated on the basis of energy, the sources of power supply are predominantly in the form of owned and operated coal-fueled generation (35%) and purchased power (63%).

Under the Utilities Group (BHUH), BHC distributes natural gas to approximately 547,300 natural gas utility customers in the states of Colorado, Nebraska, Iowa, and Kansas. Through its subsidiaries, Black Hills Nebraska and Cheyenne Light, BHE also has gas operations in South Dakota and Wyoming. Together, BHUH's four gas distribution utilities operate 645 miles of intrastate transmission pipelines, 12,088 miles of distribution mains, and 7,406 miles of gas distribution service lines. The number of customers served, sales volumes, revenues, and gross retail margins of BHUH's gas distribution utilities during 2015 are presented below in Table 1.

Table 1: OPERATING AND FINANCIAL RESULTS, 2015
GAS DISTRIBUTION OPERATIONS, BLACK HILLS ENERGY CORPORATION

Gas Distribution Operations				
Service Territory	**# of Customers**	**Sales Volumes (MMCF)**	**Revenues (000s)**	**Gross Retail Margins (000s)**
Regulated				
Colorado	78,394	8,268	$67,857	$21,933
Iowa	154,736	15,623	$133,392	$54,133
Kansas	111,203	12,221	$100,584	$47,132
Nebraska	196,990	15,024	$147,498	$64,654
Wyoming	44154	5,502	$44,161	$22,007
Total	585,477	56,638	$493,492	$209,859
Transportation	5,932	65,722	$26,506	$26,506
Non-Regulated			$31,302	$15,290

The non-regulated energy group is comprised of Power Generation, Coal Mining, and Oil and Gas segments. The Power Generation segment produces electric power from two generating plants, Wygen 1 and Pueblo Airport Generation (Pueblo). BHE's Wygen 1 generator is a mine-mouth co-owned unit, where BHE's holds 68.9MW interest, whereas Pueblo is a dual unit 200MW combined-cycle station. The two generators provide capacity and energy to the Black Hills electric utilities under mid- and long-term contracts.

The Coal Mining segment produces coal at its mine near Gillette, Wyoming, and sells low sulfur subbituminous coal primarily under long-term contracts to electric generating facilities including Neil Simpson II (Black Hills Power), Wygen II (Cheyenne Light), Wyodak (Pacific Corp, Black Hills Power), and Wygen III (Black Hills Power, MDU, and City of Gillette). In view of the long-term contractual agreements in place among its affiliate operations, BHC has essentially put in place a vertical supply chain of mining, power generation, and retail electricity services, cutting across non-regulated business lines and regulated utilities.

The Oil and Gas business segment engages in the exploration, development, and production of crude oil and natural gas, primarily in the Rocky Mountain region. The main assets include properties in the San Juan Basin (on tribal lands of the Apache Nation in Mexico and the Southern Ute Nation in Colorado), Power River Basin in Wyoming, and the Piceance Basin in Colorado. The respective reserve shares for these regions are 18%, 25%, and 50% for the San Juan Basin, Power River Basin, and the Piceance Basin.

While perhaps not the full story, the comparative size of the operating income flows across business segments reveals both the concentration of assets and, as a matter of importance, contributions of segments to the returns to capital, in total. Accordingly, shown below in table 2 are financial results from operations for BHE's utilities and non-utility operating business segments, 2013 – 2015.

Table 2: FINANCIAL RESULTS, 2013-2015 ($ million)
BLACK HILLS ENERGY CORPORATION

Revenue Flows ($ million)	2015	2014	2013
Utilities	1,231.1	13.15.1	1,205.0
Non-regulated	199.1	206.0	194.5
Other Charges	-125.7	-127.5	-123.7
Total	1,304.6	1,393.6	1,275.9
Income From Operations ($ million)			
Regulated Activities			
Electric Utilities	79.3	59.6	52.1
Gas Utilities	37.8	41.9	32.7
Total, Utilities	117.1	101.4	84.8
Non-Regulated Activities			
Power Generation	32.6	28.5	16.3
Coal Mining	11.9	10.5	6.3
Oil and Gas	-180.0	-8.5	-1.8
Total, Non-regulated	-135.4	-30.4	-20.9
Other Charges/Gains	-13.8	-1.0	12.6
Income from Ongoing Operations ($ million)			
	-32.1	130.9	117.4

The sizable operating loss reported for BHC's oil and gas in 2015 is a consequence of very large charges for impairment, reduction in the market value of proven reserves (predominantly oil). As BHE states:

> "Impairment of long-lived assets represents a non-cash write-down in the value of our natural gas and crude oil properties driven by low natural gas and crude oil prices. The write-down reflected a trailing 12 month average NYMEX price of $2.59 per Mcf, adjusted to $1.27 per Mcf at the wellhead, for natural gas, and $50.28 per barrel, adjusted to $44.72 per barrel at the wellhead, for crude oil."

The charges against income associated with the write-down of BHC's assets held as reserves has of course been driven by changes in expected prices, not observed historical prices. Capital markets have noticed. With respect to Black Hills Corporation, Value Line Investment Survey notes:

> "Black Hills' exploration and production unit is being refocused to provide gas reserves for the company's utilities under the aforementioned program. Low commodity prices have sent the E&P operation into the red and have forced Black Hills to take impairment charges in 2015."

While it is not stated explicitly, we anticipate that much of BHC's revision to its asset values is surely driven by the approximately 55-60% decline in crude oil prices, based on the WTI benchmark. Price declines this sharp can cause a large attenuation of the short- and long-term outlook for prospective prices; hence, the assets are worth significantly less in present value terms. Such write downs are of course common to the oil and gas industry for 2015, as discussed in innumerable commentary across the business news media – examples are as follows:

> "U.S. oil-and-gas producers have written down the value of their drilling fields by more in 2015 than any full year in history, as the rout in commodity prices makes properties across the country not worth drilling"…"A group of 66 oil and gas producers have taken impairment charges totaling $59.8 billion through June, according to a tally by energy consultancy IHS Herold Inc. That tops the previous full-year record of $48.5 billion set in 2008, IHS says."[4]

Observations are as follows:

- The various operations of Black Hills Corporation are strongly concentrated in gas and electric utilities, with electricity the larger. Non-utility operations are comparatively small, approximating 16% of the utilities group, in revenues.
- Within the non-regulated business group power generation and coal mining have fast rising operating income, increasing by approximately twofold over the two-year interval 2013-2015.
- BHC's oil and gas business sector has experienced an operating loss over recent years. This includes the experience of 2013, which proved to be a highly profitable year for the industry as a whole. Brief review of results for oil and gas over the 2011-13 reveal non-trivial declines from 2011 through 2013, mainly in natural gas. Gas and oil production volumes declined in 2013 from 2012, and 2011 also. Similarly, gas and oil reserves decreased over the 2011-13 period.[5]

BHE'S PROPOSED COST OF GAS HEDGE PROGRAM (COSG)

In its petition, the Company seeks authority from the Nebraska PSC with respect to the Cost of Gas Hedge Program, as follows:

[4] Ryam Dezember, "Write-Downs Abound for Oil Producers," *The Wall Street Journal*, Updated Sept. 13, 2015. Retrieved March 18, 2016.

[5] Reserves, measured in MMCFe, declined significantly in 2012 from 2011 then rose modestly in 2013.

- Granting of waivers from affiliate rules or regulations or ring-fencing commitments that the Commission deems applicable;
- Approval of revised tariff sheets and authorization for the recovery of amounts incurred under the Cost of Gas Hedge Agreement (COSG Agreement) from customers, through the Company's PGA and annual cost adjustment (AGA) clauses contained in the Company's Nebraska tariffs;
- Authorization to cover up to 50% of Black Hills Nebraska's annual sales volumes under the COSG Program; and,
- Authorization to enter into the Cost of Gas Hedge Agreement with Black Hills Utility Holdings, Inc.[6]

In its petition before the Nebraska PSC for the authority identified above, the Company clarifies the various details of the proposed Cost of Gas Hedge Program. In summary, gas exploration and production activity would be carried out by an affiliate organization of BHUH, referred to as COSGO (Cost of Service Gas Company). As proposed, COSGCO will utilize outside funding to acquire, develop, and operate gas producing properties.

Under the proposed COSG program, COSGCO does not intend to sell gas directly to the operating subsidiaries of BHUH. Rather, the natural gas produced by COSGCO will be sold in competitive wholesale markets within the Rocky Mountain region. As proposed, if the revenue flows obtained from the sale of gas by COSGCO deviate from COSGCO's all-in costs, the net gains and losses would be shared with Blacks Hills Nebraska's customers, and similarly for customers of other participating subsidiaries of BHUH. The production and sale of natural gas by COSGCO is expected to obtain net gains, in the form of revenue flows greater than costs. Net gains, and losses, are incorporated within the Purchase Gas Adjustment (PGA) clauses of the respective gas markets served by BHE through its utility holdings group, BHUH. To the degree that regional gas prices have positive covariation, the proposed structure for the COSG program includes a partial hedge: High (low) prices for gas purchased by Black Hills Nebraska would tend to be offset by positive (negative) benefits arising from COSGCO's operations.

The proposed COSG Program contains key features regarding the long-term strategy and contractual terms between BHUH's (and COSGCO, implicitly) and the gas distribution subsidiaries. First, the authorized subscription volumes under the proposed COSG, measured as percentage share of total annual retail sales, can vary across the franchise service territories within the states served by BHUH. However, Black Hills Utility Holdings must obtain authorization from the respective state regulators for a sufficient subscription volume, in total, to warrant implementation of the COSG program. Second, the structure of the all-in total costs incurred by COSGCO, which serve as the basis for determining the shared net gains and losses, are akin to that well known to privately owned utilities, and include gross and net plant, allocated corporate overheads, and weighted average cost of capital set according to recently authorized return on equity for gas and electric utilities nationally. COSGCO operations would

[6] Reference the petition filed before the Nebraska PSC by Black Hills Nebraska.

be subject to oversight by a hydrocarbon monitor, jointly agreed to by BHUH and the respectively regulatory authorities. COSGCO books of account would be subject to regular outside review by independent auditors agreed to by the regulatory authorities.

The Company's stated objective is to minimize the long-term price risk associated with natural gas purchases, as absorbed by retail gas consumers. The approach: put in place an imperfect gas price hedge, where the internal cost of gas serves as the benchmark – essentially, the strike price. In terms of strategy, COSGCO under BHUH seeks to acquire, within five years, gas production properties that result in annual gas volumes of 3 – 15 BCF annually. As proposed, COSGCO would implement drilling plans that facilitate the production of natural gas which is largely follows the subscription levels agreed to by the regulatory authorities. Because the subscription metric is a percent of annual retail sales, production levels will seemingly take account for ongoing growth in retail sales volumes, as projected.

ANALYSIS OF THE PROPOSED COSG PROGRAM

BHC's proposed COSG Program is conceptually attractive, and worthy of serious consideration by the Commission. A substantial amount of material has been presented for the consideration of the Commission in the immediate proceeding. Yet, key elements remain unresolved: how does Black Hills Nebraska intend to account for risks and uncertainty within the process of assessing candidate properties? Second, how should benefit and losses be shared; implicitly what level of risks should retail consumers assume, in view of the claim of benefits to be had, under COSG? At the very least, the tradeoff between benefits and risks needs to be discovered through an exploratory process involving, most likely, fairly intensive analytics.

Analysis Framework

The *modus operandi* of gas procurement for gas distributors involves contractual arrangements for supply covering short- and long-term forward periods. The prices for transactions under these arrangements are determined over short- to moderate-term forward periods – perhaps three years. Prices can be specific to market conditions, and it is common for gas distributors to hedge short-term price variation through some combination of forward contracts, exchange-traded futures contracts, and bilateral financial swap agreements. Gas storage also serves to hedge price variation.

As described above, the Company's proposed COSG program is a significant departure from conventional supply arrangements. As proposed, COSG set apart gas procurement for Black Hills Nebraska's in two key respects: First, the retail costs of gas are closely linked to internal costs incurred by COSGCO, not exclusively to market prices. Second, COSG involves a fairly long-term resource commitment by COSGCO and retail consumers, as COSG covers long-term forward timeframes, perhaps 20 years forward. In this respect, assessing COSG is akin to the resource analytics which underpin the resource choices of electric utilities within integrated resource plans, filed with regulatory authorities.

As mentioned by Witness Bennett, the potential commitment to BHC's COSG Program by the Commission is a matter of prudency – assessing the economic merits of the proposal. The issues at hand are fundamental: is the proposed COSG program economically viable? Second, are retail consumers likely to obtain net benefits, where the worth of the payoffs in the form of lower gas prices, are sufficient to offset risks implied by the structure of the program? To this end, it is essential to get the analytics right: understanding and measuring benefits under conditions of risk and uncertainty. This involves capturing, with computer simulation (models), the full array of factors that contribute to outcomes - essentially, the range of potential realizations of net revenue and cost flows as a consequence to COSGCO's activities over future years.

Net present value (NPV) methods, as utilized by Black Hills Nebraska in its filing before the Commission, constitutes the generally accepted analysis foundation underlying resource decisions. Without doubt, substantial payoff risk is inherent to gas exploration and production. As a consequence, BHC and the Commission are encouraged to apply NPV methods which are sufficiently expansive to take account of the various dimensions of risks facing this class of resource decisions, over extended forward time frames.

The starting point is to identify the contributing factors, including policy variables and exogenous factors – essentially, events and variables that "drive" the fundamental economics of gas properties. Policy variables are factors that are partially controlled by BHC – decision variables that assume discrete values selected from a plausible set of parameters. Exogenous factors refer to variables that are outside the immediate, direct control of BHC. Generally speaking, outside factors are well known and easily observable within historical experience. Outside factors can be viewed as random variables which can assume a range of potential values drawn from the statistical distributions of experience over historical periods; from other sources such as expectations of forward gas prices over the long-term; and from forecast services such as PIRA Group.

Contributing Factors

Contributing factors discussed above can assume the following dimensions:

Policy Variables:

- Search costs – expenditures associated with the discovery of candidate gas properties and putting contracts in place;
- Resources associated with the assessment of potential gas properties for further development;
- Capital structure and financial arrangements for funding of properties; and,
- Commitment of resources – the expenditure of capital for the purchase of rights to fields, equipment, and the potential participation in jointly-owned operations.

Exogenous Factors:

- A set of potential paths of alternative natural gas prices over future years;
- Basis point price differences, for the commercial hubs at which Black Hills Nebraska expects to purchases gas for Nebraska gas consumers, and the locations at which COSGCO anticipates the sale of gas resulting from its gas properties;

- Natural gas liquids (NGL) can assume sizable volumes and, accordingly, the analytics should be applied NGLs also.

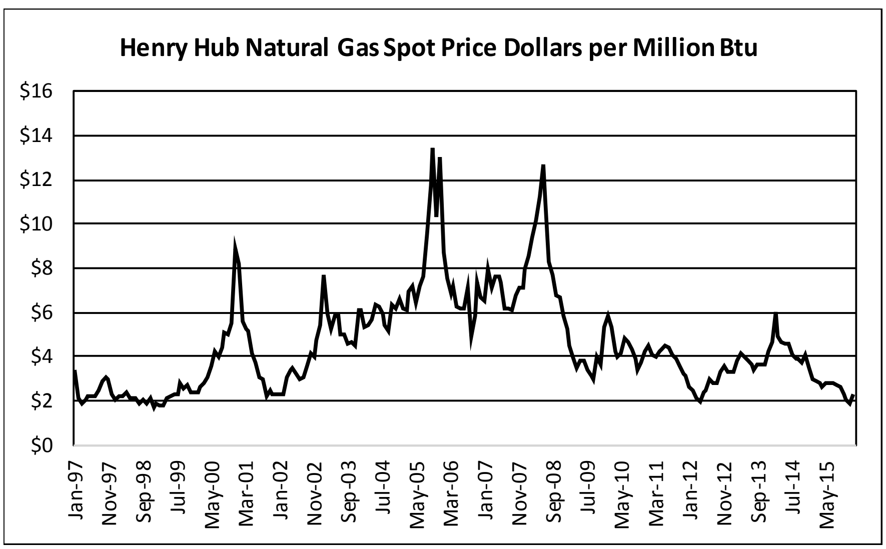
- Operations and maintenance costs associated with producing gas fields;
- Production volumes from COGSCO's properties including fields and gas wells; and,
- Capital charge rates associated with investment – rate base charges in the parlance of public utilities.

Sizing Up Risks Attending Major Contributing Factors

Natural Gas Prices:

As with commodities generally, natural gas prices can assume substantial short-term variation across days, months, seasons, and neighboring years also. Prices vary with respect to commonly recognized factors including weather, the path of macroeconomic activity, available storage capability, and occasional supply disruption. Shown below are natural gas spot prices, shown in monthly frequency.

Graph 1: HENRY HUB SPOT PRICES ($/MMBTU)



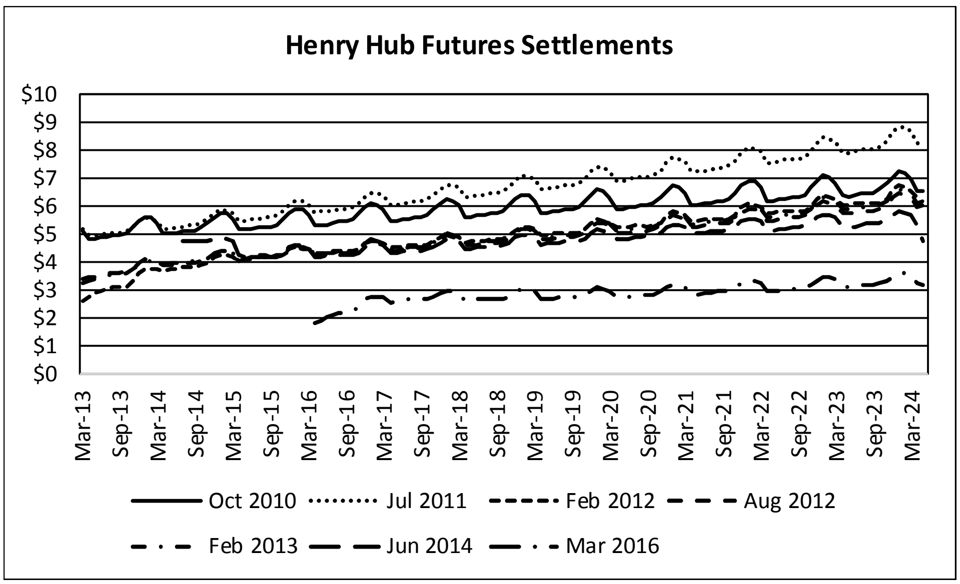
The exceptional prices experience during December of 2005 resulted from the confluence of several factors including a fast-expanding macro-economy, a comparatively warm summer, disruption in rail lines limiting the transport of coal from Power River Basin causing natural gas to be used more intensively in electricity supply, and disruption of off shore natural gas gathering lines in the Gulf of Mexico – a consequence of Hurricane Katrina. Similarly, spot prices for commercial hubs in New England reached exceptional levels as a consequence of supply shortages during early 2014 – driven once again by exceptional weather coupled with rising demand for natural gas-fueled electricity supply.

As suggested above, there is good reason for distributors to hedge to natural gas prices, particularly beyond an annual timeframe. Our limited experience suggests that, often, short-term benefits from hedging practices are less than might be expected, largely because of the structure of retail Purchas Gas

Adjustment Clauses, where variation in the purchase prices of gas are, most typically, averaged over 12-month periods. Selectively, we have found only modest gains in net benefits from short term hedge programs. However, hedge strategies reaching 2-3 years forward can provide substantially more value, providing that metrics for measuring benefits of hedge programs are interpreted narrowly: to reduce the variation in costs, absent impacts on cost levels.

As a consequence of dramatic increases in gas production and reserves, a result of technical advances (fracking methods in particular), expected gas prices have trended downward over recent years. Below are natural gas forward curves, as sampled for selected timeframes within the years 2010 forward.

Graph 2: NATURAL GAS FORWARD CURVES FOR RECENT YEARS ($/MMBTU)



The above prices are shown in nominal dollars, and it is appropriate to mention that these natural gas forward curves reflect a timeframe wherein expected long-term inflation was declining, though not in a major way. Moreover, reductions in expected demand has caused prices to mitigate somewhat. Beginning in mid-2015, a general consensus has set in: the outlook for U.S. and worldwide economic activity over the long-term is likely to trend somewhat lower than reflected in expectations held previously. For the U.S., long term real output is likely to expand at approximately 2.1% over the long term.

The counterbalance to rising supply and attenuation in domestic demand is expansion of natural gas exports in the form of LNG. To this end, numerous LNG terminals are in the works along the Eastern Atlantic and Gulf Shores of the U.S.[7]

[7] A selection of North American LNG Export Terminals proposed to the Federal Energy Regulatory Commission include the pending applications of Coos Bay, OR: 0.9 BCFd (Jordan Cove Energy Project); Astoria, OR: 1.25 BCFd

The issue of the appropriate basis for projections of natural gas prices has surfaced in the immediate proceeding. On this question, researchers at the International Monetary Fund (IMF)[8] have recently explored alternative forecast methods for various commodities, using formal statistical tests. The conclusions of this technical work favor futures prices. As stated:

> "First, futures price-based forecasts are hard to beat. Futures prices perform at least as well as a random walk for most commodities and at most horizons and, in some cases, do significantly better"…"We do not find a significant difference in the forecasting ability of futures markets during bull and bear markets, defined as when spot prices are trending higher or lower."

Further to this issue, the discussion notes that the accuracy of futures price-based forecasts declines, as the forward interval – number of years forward – increases in length. This decline in forecast performance of forward spot prices, by futures markets, is likely to follow from declining market liquidity as the forward market extends (elsewhere, Mr. Charles Loomis has advanced this view.)

Capital Risks:

It is perhaps useful, for general perspective and background, to describe the inherent business risks of the gas and oil exploration and production industry. Accordingly, shown below in Tables 3 and 4 are the realized market returns to equity investment in gas and oil, shown in contrast with gas distribution.

Table 3: MARKET RETURNS FOR OIL AND NATURAL GAS EXPLORATION AND PRODUCTION

Returns to Equity Capital, 2003--2012 (%)		
	Average Return	**Standard Deviation**
S&P 500	9.74	16.21
SIC Composite	20.71	33.43
Large Composite	20.39	33.42
Small Composite	3.41	66.62

Ibbotson Cost of Capital 2013 Yearbook, Morningstar, Inc., 2013.
SIC Code 131 - Crude Petroleum and Natural Gas (146 Companies)

(Oregon LNG); Elba Island, GA: 0.35 BCFd (Southern LNG Company); and the projects in a pre-filing status including Plaquemines Parish, LA: 1.07 BCFd (CE FLNG); Plaquemines Parish, LA: 0.30 BCFd (Louisiana LNG); and Robbinston, ME: 0.45 BCFd (Kestrel Energy – Downeast LNG).

[8] Reference David Reichsfeld and Shaun Roache, *Do Commodity Futures Help Forecast Spot Prices?*, International Monetary Fund Working Paper (WP/11/254), November, 2011.

Table 4: MARKET RETURNS FOR GASI DISTRIBUTION

Returns to Equity Capital, 2003--2012 (%)		
	Average Return	**Standard Deviation**
S&P 500	9.74	16.21
SIC Composite	13.48	15.58
Large Composite	11.54	12.81
Small Composite	18.90	27.66
Ibbotson Cost of Capital 2013 Yearbook, Morningstar, Inc., 2013.		
SIC Code 4924 - Natural Gas Distribution (10 Companies)		

Statistical variation (standard deviation) in market returns demonstrate overall risks to shareholder capital. Variation is in part[9] a consequence of inherent business/operating risks manifested in operating income, as well as financial leverage. From above, one cannot escape a key conclusion: the overall risks associated with gas distribution are significantly below that of oil and gas. Alternative risk metrics obtain similar results.

The above analysis does not reveal the full story, however. Historical variation in market returns account for risks in cash returns as well as changes in the prospects for the business perceived by capital markets. Unique to oil and gas, however, is the substantial variation in the value of reserves, capitalized on the books of account. The book value of reserves can change abruptly up or down, contributing significantly to capital market risks of the respective entities. Importantly, while changes in the value of reserves are relevant to the future prospects of the relevant oil and gas business entities, changes in reserves have little to do with current and year-over-year variation in operating performance and internal cash returns to capital.

Model Structure for Assessment of BHC's COSG Program

Two types of model within the realm of the NPV framework are worthy of consideration by BHC and the Commission, including so-called *fully dimensioned* discounted cash flow (DCF) and *real option value* (ROV). These two methods are well suited to internalizing various dimensions of risks within the process of valuation.

Dimensioned DCF augments conventional static DCF by incorporating a sufficiently dense set of potential values that the various contributing factors can assume, over future timeframes. Implementation of the expanded DCF approach may require a substantial amount of statistical analysis of historical experience, particularly with respect to key factors including regional prices for natural gas and natural gas liquids,

[9] Variation in market returns for individual listings and industries are, in large part, a result of variation in equity markets as a whole. For this reason, we include the experience of the S&P 500 for reference.

basis point price differences across relevant commercial gas hubs, production quantities of acquired gas properties, and operating costs.

Critically, some contributing factors listed above are likely to be highly correlated. Hence, it is appropriate to consider the development of a joint distribution (Gaussian Joint Distribution (GJD) of the statistical variance and covariance of the relevant factors, and then randomly drawing from the GJD to populate the DCF model with sets of alternative values, for the relevant factors. The statistical analysis used to determine the GJD should be conducted in monthly frequency and take account of auto correlation. Where relevant, the results of these analyses can be used in conjunction with projections of gas prices obtained from outside forecast services. Generally speaking, it is likely that prices for natural gas and NGLs are correlated. Less clear is the relationship between production quantities and operating expenditures, for operating fields and wells.

Real Options Valuation explicitly accounts for the options embedded in, or attached to, physical or real assets – thus the notion of *real options*. Real options are distinct from options relating to financial assets – securities and other financial claims. ROV is a process by which a real or tangible asset with numerous uncertainties can be valued in a coherent manner when flexibility – or potential options – is inherent to the asset, such as gas properties, or required for investment decisions.[10]

As mentioned above these more expansive methods, are essential to the evaluation of potential gas properties, particularly in view of the comparatively high levels of risks inherent to gas exploration and production. Within oil and gas, it appears that these two valuation methods have been well in play for some time, with real options valuation assuming the more prominent role. In the early 1990s, Houston-based Anadarko Petroleum Corporation (Anadarko) outbid competitors for the Tanzanite block in the Gulf of Mexico. It found oil and gas there in 1998 and was producing within three years. The Tanzanite discovery is significant not so much for oil and gas abundance, but that in bidding for it, Anadarko broke with general industry practices. Rather than using only conventional discounted cash flow (DCF) analysis in support regarding the worth of the Tanzanite block and how much to bid for the lease, Anadarko applied real-options valuation (ROV). ROV gave Anadarko the confidence to outbid others because the analysis suggested that there was more value to the Tanzanite lease than DCF analysis implied. Along with the rest of the players in the oil and gas industry, Anadarko routinely uses ROV methods as the basis for investment and resource decisions.

ROV is not necessarily a replacement for conventional DCF methods of valuing potential investments. Most oil and gas companies still use DCF and Internal Rate of Return (IRR), which are variants within the Net Present Value framework, to appraise potential investments. These conventional NPV metrics have

[10] "In today's extremely turbulent world, managers recognize how risky the most valuable investment opportunities often are, and how useful a flexible strategy can be", as stated on the cover jacket of *Real Options: Managing Strategic Investment in an Uncertain World*, by Martha Amram and Nalin Kulatilaka, Harvard Business School, 1999.

served and continue to serve an important role in asset valuation for purposes of investment decisions. However, the applications are sophisticated – i.e., *fully dimensioned* – thus incorporating algorithms to explicitly account for risk associated with the relevant contributing factors, as listed above.

More specifically, ROV has been shown to provide more exact valuations than conventional DCF methods, primarily because the approach more closely reflects real world variation and uncertainty associated with the contributing factors discussed above. ROV can often highlight value in potential projects that traditional methods such as DCF may be unable to reveal.

ROV is often used as a complement to the traditional methods – indeed, DCF can be integrated within the technical calculations which obtain ROV results. In brief, ROV combines and integrates the best of scenario planning, portfolio management, decision analysis, and option pricing.

Conventional discounted cash flow analysis, an application of the NPV paradigm, is relatively straightforward: a prediction of a steam of cash flows (revenues minus costs) over the expected life of the asset (or a project such as an oil and gas field) are discounted at a capital cost rate (e.g., the weighted average cost of capital) to reflect both the time value of money and the inherent riskiness of those cash flows. The time value of money reflects the fact that money held in the future is worth less than money held at the present, as money in hand can be invested and earn interest, whereas money in the future cannot.

As mentioned, the pivotal component of DCF is Net Present Value (NPV) – the present value of cash inflows (revenues) minus the cash outflows (costs) over the life of the asset (or investment). A positive NPV indicates the asset creates value; a negative NPV indicates the asset destroys value. Thus, the NPV decision criteria: invest in project (asset) wherein expected NPV is positive; avoid projects wherein NPV is negative. Thus, DCF can be described as a clear, consistent decision framework. However, precisely because of its clarity, DCF has limitations insofar as it cannot readily account for risks and uncertainty – at least as conventionally formulated.

DCF is a static and comparatively inflexible decision framework. Typically, DCF is applied under an assumption that a project plan is frozen and unalterable, and that project management is passive and follows an original plan regardless of whether circumstances change that may alter the value of the project prospectively. This is unrealistic: project managers modify plans as circumstances change and as uncertainties are resolved about particular variables relevant to the project's value (e.g., output prices, input costs). As a result of learning over time, management decisions about the project path adds value not captured by DCF. Thus, conventional DCF analysis or variants of it fail to take account of intrinsic attributes of an asset or investment opportunity.

Several potential problem areas arise in applying DCF calculations to strategic optionality: undervaluing an asset that currently produces little or no cash flow, the variability over time of the weighted average cost of capital discount rate, the estimation of the asset's economic life, forecast errors in estimating the future cash flows, and insufficient tests for the plausibility of the final results. Real option value methods can selectively mitigate some of the DCF limitations; however, ROV methods also have limitations.

Real Option Value: A real options (ROV) approach is a learning model, such that management makes better and more informed strategic decisions[11] as uncertainty with respect to the contributing factors are resolved through the passage of time. ROV can be viewed as dynamic augmentation to the static and inflexible approach to classic DCF analysis where resource decisions are made initially with no recourse to choose other pathways or options in the future.

Simply defined, real options valuation is a systematic approach to obtain an integrated solution using financial theory.[12] ROV provides the basis to integrate economic analysis, management and decision sciences, and statistics and econometric modeling into a common framework that is fundamentally DCF.

The word *option* implies added value. For example, when one hears the expression "keeping options open", having more than one option and not foreclosing on any options, the underlying implication is that holding an option usually has value whether or not it is exercised. The same is true of real options – options that arise from flexibility with respect to holding real physical assets.

ROV draws heavily from financial options theory[13] to assess real physical assets in a dynamic and uncertain business environment in which business decisions are flexible in the context of strategic capital investment decision-making. Financial options are derivatives, where value is *derived* from other underlying financial assets such as shares of common stock, or debt instruments. Real option valuation applies the thinking behind financial options to evaluate physical (real) assets. By analogy with a financial option, a real option is the right, but not the obligation, to take an action affecting a real physical asset at a predetermined cost for a predetermined period of time – the life of the option.[14]

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[11] A useful reference is the work of Lenos Trigeorgis, *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, MIT Press, 1996.

[12] Reference Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty*, Princeton University Press, 1994.

[13] The classic reference for option pricing is Fischer Black and Myron Scholes, *The Pricing of Options and Corporate Liabilities*, Journal of Political Economy, 1973. The result of this publication, referred to as the Black-Scholes model, gave rise to a number of expansions including, in particular, risk-neutral pricing.

[14] Generally speaking, a financial option is the right, but not the obligation, to buy or sell an asset at (or sometimes before) a particular date at a particular price. The price at which a share can be bought or sold, if the option holder chooses to exercise the right, is known as the *exercise price* or the *strike price*. The two main types of options are the *call option* – to buy the share at the strike price – or the *put option* – to sell the share at the strike price. If the share price exceeds the strike price at the point in time at which the option can be exercised, a call option is said to be *in the money*. If the share price fails to reach the strike price, the option is said to be *out of the money*. Generally speaking, financial options can be subdivided into several types, the most common of which are the European option and the American option. A European option can be exercised only on the expiration date specified in the option contract. An American option may be exercised at any time up to and including the expiration date. Options have two important features. First, they give the holder the possibility of a large upside

POSITIONS OF INTERVENING PARTIES

The positions of parties to the immediate proceeding are summarized as follows:

Constellation NewEnergy (CNE Gas Holdings, LLC)

Witness Sorenson

Mr. Sorenson's recommendation calls for the Commission to deny BHE's COSG Program as proposed. Mr. Sorenson expresses the view that the COSG program has the potential to reduce or remove the freedom of natural gas customers served through the competitive market in Nebraska to choose their natural gas supplier.[15] Mr. Sorenson's recommendation also incorporates the reasons articulated in the testimony of Mr. Stephen Bennet, also filed on behalf of Constellation NewEnergy.

Witness Bennett

Mr. Stephen Bennett's testimony articulates the mechanics of the Company's proposed COSG program, and is worthy of careful review. In particular, Mr. Bennett highlights the risk transfers inherent to the COSG, succinctly stating:

> "There is great efficiency to be gained by procuring natural gas through market mechanisms like competitive wholesale markets and gas choice that allocate resources in the most effective manner…The Application and the COSG Program undermine the logical, separate treatment of distribution and commodity assets by using guaranteed ratepayer cost recovery to acquire natural gas reserves under COSGCO and by effectively passing a guaranteed profit on those reserves through to those ratepayers via the PGA."

Witness Bushra

Mr. Bushra's testimony discusses the hypothetical COSGO Model supplied by BHC to assess the value of the COSG Program to Nebraska customers. Mr. Bushra describes how the model is highly sensitive – in terms of whether it results in a positive outcome for Nebraska customers – to the quality and accuracy of the forecasts it uses as inputs. Mr. Bushra also notes that the model appears to incorporate favorable assumptions regarding natural gas prices that do not reflect market conditions. As a result, Mr. Bushra concludes that the

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gains while protecting against a downside risk. Second, they are more valuable the higher is the uncertainty or risk associated with the investment decision. Therefore, option contracts have real value and carry a cost to obtain.

[15] While not explicitly stated, we infer that Mr. Sorenson's concerns are that the COSG program may convey an unfair cost advantage to BHUH's Nebraska operations, causing Black Hills to be more competitive within the respective Nebraska markets subject to supply competition. We generally concur: potentially, COSG may convey measurable cost advantage to the Company in Nebraska's competitive choice markets. We do not find the potential realization of any such advantage to be unfair.

positive results for Nebraska customers shown by the model supplied by BHC require additional scrutiny.

Public Alliance for Public Alliance Community Energy (ACE)

Witness Ackland

Ms. Beth Ackland requests that the Commission temporarily suspend Black Hills Nebraska's application. ACE is a cooperative natural gas wholesale purchaser, operating on behalf of a number of comparatively small Nebraska municipalities. The underlying role of ACE is to execute strategies "to ensure competition in the Nebraska Choice Gas Program and to provide the best possible pricing for end-use natural gas customers."

The testimony of Ms. Ackland is focused on the potential impact of the proposed COSG program on prices. Ms. Ackland's view is that the Company's application is incomplete: the Company's filing does not include information with respect to key dimensions of BHE's resource plans. Ms. Ackland recommends that the Commission request BHUH, through Black Hills Nebraska, provide the Commission with a thorough review of its plans to combine operations of its existing and acquired properties. Ms. Ackland's view appears to be that, absent this information, the Commission's decision process with respect to the Company's application cannot fully compare and contrast the benefits and risks to ratepayers of the proposed COSG program, compared to the expansion of the Nebraska Choice Gas Program into existing BHUH services territories across the region. Mr. Ackland recommends, at a minimum, at least one year of Choice Gas Program data should be evaluated and submitted by BHUH to the Commission, before determining which program could provide the most benefit to ratepayers.

Nebraska Municipal Power Pool ("NMPP")

Witness Harms

Mr. John Harms' recommendation calls for the Commission to evaluate and understand alternative hedging strategies, in conjunction with its review of the Company's COSG program. In Mr. Harms' view, alternative hedging strategies could be submitted by BHUH or energy consulting firms, and should include a review of hedging programs that have been adopted by regulatory authorities in other states. Mr. Harms also recommends that the Commission consider that some gas customers may prefer to have the cost of gas purchases, reflected in the Company's PGA, to reflect market prices and have based their previous decisions to install natural gas consuming systems on their own evaluation of natural gas costs relative to other alternatives, and that such consumers likely should not be forced into acceptance of a price managed program.

Public Advocate for the State of Nebraska

Witness McGarry

Mr. Michael McGarry's recommendation is for the Commission to deny the COSG Program as proposed because it would not be in the public interest based on the following factors:

i. The proposal unduly shifts the risk of excessive costs and inappropriately guarantees cost recovery of an unregulated affiliate's investment and operating costs through the hedge true-up.

ii. The proposed method to establish the return of investment for the investments in the development of the reserves is unreasonable and could lead to ratepayers providing a higher return than they currently pay.

iii. The requested debt/equity ratio of 40/60 will overstate the return to the unregulated affiliate COSGCO.

iv. The 60-day review period under the proposal is too short to provide adequate review considering the volume of data and the cost implications.

v. The brief review period also applies to forecasts, unduly shifting risk to ratepayers that traditionally and naturally should be borne by the company.

vi. The Company's plan to use a Hydrocarbon Monitor and an Accounting Monitor who would be hired and paid for by the Company, with only approval by the Commission, is an inadequate independent safeguard to provide the necessary expert evaluation for the Commission and intervenors.

vii. The hypothetical/illustrative example provided in Company Exhibit AC-2 combined with the sensitivity of that analysis to any changes in the underlying assumptions, makes it virtually impossible to know when, and even if, customers will start to see benefits from the COSG program.

viii. If one or more commissions/utility boards of BHUH Utilities do not approve the similar COSG program in their respective jurisdictions, the remaining operating companies will be left to shoulder the burden of the costs that would have been allocated to and paid for by the other operating company(ies) not receiving approval for the COSG program.

ix. The Termination Clause of the COSG Agreement may usurp the Commission's ability to ensure just and reasonable rates for Black Hills/NE customers.

RECOMMENDATIONS AND CONCLUDING COMMENTS

Recommendations

The immediate proceeding has covered much ground, so far. Together, the Company's filing, an intensive and highly focused discovery process involving a substantial level of material, and intervenor testimony and accompany recommendations provide the Commission with a close look at the workings of BHC's COSG. While we find Black Hills Nebraska's COSG program conceptually viable, key empirical issues remain unresolved by the record. As a consequence, we recommend that the Commission set aside BH Nebraska filing and proposed COSG for the time being, and immediately initiate a subsequent proceeding or process focused on reaching resolution to two fundamental issues:

1. *Resource Viability:* Demonstration of the viability of the Company's COSG program with quantitative analysis. Black Hills Nebraska has not convincingly demonstrated, with sufficient surely, that the acquisition of gas properties by COSGCO will provide net benefits to retail gas consumers in Nebraska. In a subsequent proceeding/process, the Commission should expect Black Hills Nebraska to:
   - document the quantitative methods that it intends to employ, for purposes of evaluating potential properties;
   - provide example analytics that amply demonstrate how properties will be evaluated. In view of the Company's considerable experience with natural gas activities including discovery, development, and production, the Company should assemble a representative sample of properties, drawn from this history or other sources as appropriate; and
   - demonstrate how it intends to quantitatively assess risks and uncertainty, covering the various contributing factors, as listed earlier.
     - Using the set of relevant characteristics and attributes of the sample of properties, we anticipate that, in its filing, the Company will take account of the full range of risks and uncertainties, such that the Commission and stakeholders can understand the magnitude involved: the range of potential payoffs (i.e., net benefits) and risks, measured in monetary terms.
2. *Benefit and Risk Sharing:* Drawing upon the analysis inherent to 1), the Commission and stakeholders should explore alternative arrangements for sharing risks and benefits between retail consumers and BHC under the proposed COSG.

Absent the Company's agreement to participate in the subsequent proceeding outlined above, the Commission should set aside further consider of the Company's COSG, as currently proposed.

Comments

BHC's COSG Program implicitly highlights a fundamental question: Is market competition within the natural gas exploration and production industry sufficiently intensive to drive prices to levels that approximate all-in costs? If the answer is in the affirmative, wholesale markets prove to be workably competitive, by definition: Actions by individual producers – i.e., changes in price offers or quantities supplied – do not translate into sustainable above-normal profit levels.

The conventional view certainly holds that gas exploration and production is highly contestable. Barriers to entry do not appear to be particularly high vis-à-vis other industries. This suggests that opportunities for net gains ensure that potential gas properties are discovered and developed. The presence of competitive gas supply implies that the COSG program can obtain net gains for retail consumers in two ways, only:

- *Discovery and acquisition* by COSGCO of gas properties that are underpriced. That is, recoverable reserves inherent to the acquired properties are not fully capitalized in the market worth.

- *Production and capital costs*[16] of COSGCO have comparative advantage with respect to that of other gas producers – gas markets as a whole.

Conventional wisdom and various capital market risk metrics, including statistical variation in operating income, suggest that gas exploration and production is a comparatively high risk industry. Inclusion of COSG within Nebraska's retail PGA presumes that consumers assume this capital risk. However, if the local market price realizations associated with gas sales by COSGCO move in high correlation with that of the prices paid by Black Hills Nebraska, the capital risks inherent to gas exploration and production are implicit in the hedge features of the COSG, and thus offset. This feature of the proposed COSG Program is akin to weather hedge programs, such as Georgia Power's fixed bill program.

The COSG Program is arguably not so much a hedge to mitigate gas price variation, as a potential vehicle to obtain lower overall gas prices – essentially, a lower overall path for retail gas prices paid by consumers prospectively.

Because the potential purchase of gas properties by COSGCO are intended to serve retail consumers, the Commission and Black Hills Nebraska may wish to consider, within the analytics, the use of a consumer discount rate in lieu of the weighted average cost of capital associated with COSGCO. An example calculation is presented in Appendix 2.

For the hedge features of the COSG to be effective, it will be necessary for the underlying term (months) of the transactions for the sale of gas produced by COSGCO, to be closely align with the term for the gas purchases by Black Hills Nebraska, for retail markets.

It would appear that, should COSGCO assume possession of properties of Black Hills Exploration and Production (BHEP), BHUH's retail gas distribution business will take on the characteristics of a vertical supply chain, similar to that of its retail electric utility business.

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[16] We speculate that the capital cost rates (discount rates) used to assess candidate properties by COSGCO could be markedly below that of competitors: Under COSG, COSGCO's discount rate is that of a regulated utility whereas that of competitors is an after-tax weighted cost of capital, including risk-adjusted cost of equity. Note that, for oil and gas exploration and production, the Capital Asset Pricing Model (CAPM) Betas are significantly above unity, whereas CAPM Betas for electric and gas utilities are significantly less than unity, stated with or without Blume or Vasicek adjustments.

APPENDIX 1

DETERMINING ASSET WORTH USING REAL OPTIONS VALUATION

As presented within the body of the report Real Options Valuation (ROV) allows managers to evaluate real options to add value to firms, by giving managers a tool to recognize and act upon opportunities to amplify gain or to limit (or mitigate) loss. The profession literature of financial economics and financial valuation contains numerous discussions, including example analyses. In particular, two academicians, Michael Brennan and Eduardo Schwartz[17], provide an example of valuing the rights to operate a gold mine.

In the example analysis of Brennan and Schwartz, a mining company owns the rights to a local gold mine. The rights provide the firm with the option, and the legal obligation, to mine the gold reserves that are supposedly abundant in the mine. Therefore, if the market price of gold is high, the firm may decide to start mining, but if the market price of gold drops, the firm may decide to stop and wait for a later time to begin mining. If the cost of mining is C and payoff on mined gold is S, accounting for the time value of money, then the payoff schedule can be written as:

S – C	if and only if	S > C
0	if and only if	S ≤ C

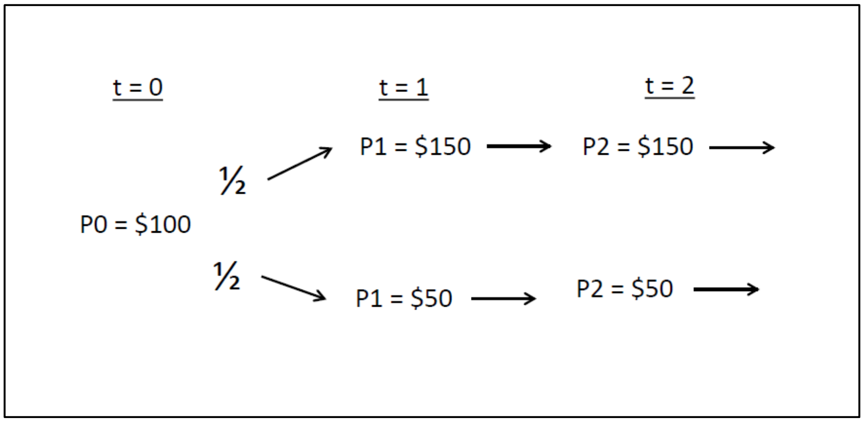
This payoff schedule is identical to a payoff schedule on a call option on the underlying asset, the value of the mined gold. If the cost exceeds the value of the underlying asset, the option is left to expire as worthless, without execution; otherwise, the option is exercised.

The gold mine example illustrates the main idea underlying ROV: an investment decision can be treated as the exercising of an option. The firm has an option to invest. It need not exercise the option now, but can wait for more information to resolve uncertainties. When investment is irreversible (i.e., there is a sunk cost), there is an opportunity cost of investing now rather than waiting. Opportunity cost (i.e., the value of the option) can be very large. The greater the uncertainty, the greater the value of the firm's options to invest, and the greater the incentive to keep these options open. Note that the value of a firm is the value of its capital in place plus the value of its growth options. Any investment decision involving sunk costs can be (and probably should be) viewed as an option.

Another example will illustrate the elements of the use of real option valuation. Presume a company is considering building a widget factory that will produce one widget per year forever. The market price of a widget now (period 0) is $100, but next year (period 1) the price may go up or down by 50%, and thereafter remain constant (period 2, 3, …). The price path can be represented by the following schematic in Figure 1:

[17] Brennan, M. J., and E. S. Schwartz, "Evaluating natural resource investments", *Journal of Business*, 58, pp. 135-157, 1985; and Brennan, M. J., and E. S. Schwartz, "A new approach to evaluating natural resource investments", *Midland Corporate Finance Journal*, pp. 37-47, 1985.

Figure 1 Price Path for Investment Example with Option



Assume the cost to build the factor is $800, and it only takes a week to build it. Is this a good investment? Should the company invest now or wait one year to see whether the price goes up or down?

If the company invests now, the net present value (NPV) of the investment is given by equation 1.

$$NPV = -\$800 + \sum_{t=0}^{\infty} \frac{\$100}{(1.1)^t} = -\$800 + \$1{,}100 = \$300 \qquad (1)$$

Equation 1 says that the NPV is $300, which would indicate the company should invest now in the widget factory. However, suppose the company decides to wait one year and then invest if the price goes up. In that case, the NPV is given by equation 2.

$$NPV = (0.5) \times \left[\frac{-\$8\ 00}{1.1} + \sum_{t=1}^{\infty} \frac{\$15\ 0}{(1.1)^t}\right] = \frac{\$4\ 2\ 5}{1.1} = \$386 \qquad (2)$$

The decision to wait to see if the market price of widgets increases provides greater value than to invest now. The value of being able to wait is $386 minus $300 or $86. The real option value, that is, the value of having the flexibility to invest in the widget factory now or one year from now, is $86.

Another way to value flexibility is to answer the following question: How high an investment cost (*I*) – e.g., the cost to build the factory – would the company be willing to accept to have the flexibility to invest now or in the future rather than be faced with a "now or never" choice? The answer involves finding the investment cost value *I* that makes the net present value of the widget factory when the company waits a year equal to the NPV when *I* = $800 and the company invests now. The NPV when the company builds the plant now is $300, from equation 1. So, substituting *I* for $800 in equation 2 and substituting $300 for $386, we can solve equation 3 for I.

$$NPV = (0.5) \times \left[\frac{-I}{1.1} + \sum_{t=1}^{\infty} \frac{\$15\ 0}{(1.1)^t}\right] = \$300 \qquad (3)$$

Solving for *I* in equation 3 yields *I* = $990. This means that for the company to build the factory now (and only now) at a cost of $800 has the same value as the opportunity to wait one year and build the factory now or next year at a cost of $990.

This example shows how using the essential tool of the DCF approach, NPV, the real option value can be determined. However, this is not the real option approach. If a real option pricing approach is applied, then the company considers that next year, if the price of widgets rises to \$150, the company exercises its option by paying \$800 and receives an asset whose value is determined by equation 4.

$$V_1 = \$1{,}650 = \sum_{t=0}^{\infty} \frac{\$15\ 0}{(1.1)^t} \qquad (4)$$

On the other hand, if the price falls to \$50, investing now in the factory, would produce an asset whose value is only \$550 (solve equation 4 by substituting \$50 in place of \$150). Since \$550 is less than the investment cost of the plant of \$800, the company would not exercise the option. The important question for the company is: What is the value of the option. This problem is solved by the real option value approach.

Let F_0 represent the value today of the investment opportunity in the widget factory. Let F_1 represent the value next year of the investment opportunity. If the market price for widgets rises to \$150, then *F1* can be determined by equation 5.

$$F_1 = V_1 - \$800 = \sum_{t=0}^{\infty} \frac{\$15\ 0}{(1.1)^t} - \$800 = \$850 \qquad (5)$$

If the price of widgets falls to \$50, the option to invest will go unexercised, so that F_1 will equal \$0. All the possible values of F_1 are known. Now the problem is to find the value of F_0, which is the value of the investment option today. To solve this problem, imagine creating an investment portfolio that has two components: the investment opportunity itself, and a certain number of widgets. The number of widgets should be that which makes the portfolio risk free (i.e., find the number of widgets *N* such that an investor in the portfolio is indifferent to a change in the market price of widgets).

Consider a portfolio consisting of the investment opportunity, and a short sale of N widgets. The value of this portfolio today is given by equation 6.

$$Q_0 = F_0 - N \times P_0 = F_0 - 100 \times N \qquad (6)$$

The value of the portfolio next year,

$$Q_1 = F_1 - N \times P_1, \qquad (7)$$

depends on P_1. If P_1 equals \$150, then F_1 equals \$850, and

$$Q_1 = \$850 \quad - N \times \$150 \qquad (8)$$

If P_1 equals \$50, then F_1 equals \$0, and

$$Q_1 = -N \times \$50 \qquad (9)$$

Now choose *N* so that the portfolio is risk free, that is, choose *N* so that Q_1 does not depend on what happens to price in period 1. This is accomplished by setting equation 8 equal to equation 9 and solving for *N*.

$$\$850 - N \ \times \$150 = \ -N \times \$50 \qquad (10)$$

The solution is N = 8.5 widgets. Substituting N = 8.5 into equation 8, Q_1 equals a minus \$425, regardless of what happens to the price in period 1.

Now calculate the return to the investor from holding this portfolio. The return is equal to the capital gain, Q_1 minus Q_0, minus any payments that must be made to hold the short position. Since the expected rate of return on a widget is zero (i.e., the expected price next year is \$100, the same as the price this year), no rational investor would hold a long position unless he could expect to earn at least 10 percent. Therefore, selling widgets short will require a payment of 0.10 x P_0 equals \$10 per widget per year. The portfolio has a short position of 8.5 widgets, so it will have to pay out a total of \$85 to hold that short position. Therefore, the return from holding this portfolio over the year is determined in equation 11.

$$\begin{aligned} Q_1 - Q_0 - \$85 &= Q_1 - (F_0 - N \times P_0) - \$85 \\ &= -\$425 - F_0 + \$850 - \$85 \\ &= \$340 - F_0 \end{aligned} \tag{11}$$

This return is risk free, so it must equal the risk-free rate, 10 percent, times the initial portfolio value, which is $Q_0 = F_0 - N \times P_0$. Thus solving for F_0 in equation 12:

$$340 - F_0 = 0.10 \times (F_0 - \$850) \tag{12}$$

F_0 equals \$386, which is the value of the opportunity to build the factory now or next year. This is the same value obtained from equation 2 using the NPV approach.

The example illustrates the options valuation approach. The evaluation depends on the initial price as well as the risk-free interest rate. The investment opportunity may involve several different stages at which a decision to proceed or not must be made, and therefore valuation may be contingent on a number of variables about which there is uncertainty.

The ROV approach is applied to such decisions about whether to defer a decision, to wait and see, to delay, to expand, to contract, to choose, to switch resources, or to engage in a phased and sequential investment process. The application of ROV technique thus leads to consideration of some fairly complex decisions over time that require an understanding of or assumptions about the probability distribution of various variables, such as the price of widgets over time in the above example. The more complex real option valuation problems that are addressing investment decisions that have a large number of dimensions lead to complex decision tree type representations (which may involve the use of binomial lattices) and, for high-dimensional problems, the analysis may require the application of Monte Carlo simulations to elucidate the volatility of the investment value over time. In very many cases, the real options valuation problem is so complex that it cannot be solved exactly or, for that matter, precisely. These types of real option valuation problems in some cases can be addressed through the application of dynamic optimization methods (i.e., the application of some form of the Bellman principle of optimality[18]).

[18] The Bellman principle (Bellman equation) is named after Richard Bellman and often referred to as a *dynamic programming equation*, is a necessary condition for optimality associated with mathematical optimization, dynamic programming. The Bellman equation is relevant to ROV because it expresses the value of a decision problem at a certain point in time in terms of the payoff from some initial choices and the value of the remaining

APPENDIX 2

Presented below is an example of a calculation of the consumer discount rate for yearend 2012. Not to be confused with a societal discount rate, a consumer discount rate is the opportunity cost of capital, stated on a weighted average basis. As shown, the consumer discount rate is estimated as the expected return on financial assets held by U.S. households,[19] equal to 7.27%.[20]

Financial Assets of U.S. Households and Cost Rates, 4th Qtr of 2012				
	Amount ($ Trillions)	**Share**	**% Rate**	**W % Rate**
Foreign depostis	50.7	0.001	0.42	0.000
Checkable deposits	814.4	0.015	0.05	0.001
Time and savings deposits	7,070.3	0.132	0.27	0.036
Money market fund shares	1,110.2	0.021	0.12	0.002
U.S. savings bonds	182.4	0.003	3.50	0.012
Open market paper	28.6	0.001	0.16	0.000
Other Treasury securities	853.6	0.016	3.59	0.057
Agency debt	73.1	0.001	2.46	0.003
Municipal securities	1,678.8	0.031	3.60	0.113
Corporate and foreigh bonds	2,325.8	0.043	3.83	0.167
Corporate equity	9,770.5	0.183	11.33	2.068
Mutual fund shares	5,300.9	0.099	7.59	0.752
Life Insurance	1,230.6	0.023	6.83	0.157
Pension Funds	14,060.7	0.263	8.34	2.192
Non-Corporate Equity	8,079.1	0.151	11.33	1.710
Miscellaneous, Other	896.7	0.017	7.27	
Total:	**53,526.4**	**1.000**		**7.27%**

The consumer discount rate reflects the expected interest rate (rate of return) on the aggregate savings held by households in financial assets, which would seemingly be the implicit cost rate on the share of income committed to savings, in lieu of current consumption. Without doubt, the underlying cost rate is highly specific to individual households and income groups.

decision problem that results from those initial choices. This breaks a dynamic optimization problem into simpler sub-problems according to Bellman's *Principle of Optimality*.

[19] The determination of the forward-looking expected return on financial assets held by households involves a number of support calculations and assumptions, and are subject to challenge. Nonetheless, the results shown herewith obtain a net real rate of return on capital of approximately 5%, which is closely approximated by other widely accepted studies including, most notably, Thomas Piketty's recently published treatise regarding the long-term return on capital across developed economies, *Capital In the 21st Century*.

[20] The various interest rates shown above (2012) undoubtedly overstate interest rates and expected rates of return on capital currently. Accordingly, any plausible estimate of the consumer discount rate is lower today, both in real and nominal terms, than during 2012.